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MISCELLANEOUS.

Conducted by J. M. COLAW, Monterey, Va. All contributions to this department should be sent to him.

SOLUTIONS OF PROBLEMS.

30. Proposed by R. J. ADCOCK, Larchland, Warren County, Illinois.

When the sum of the distances of a point of a plane surface, from all other points, is a minimum, that point is the center of gravity of the plane surface.

IV. Discussion by O. W. ANTHONY, M. Sc., Professor of Mathematics in New Windsor College, New Windsor, Maryland.

I. Consider the following problem: Find a point within a plane surface such that the sum of the n^{th} power of the distances to all other points of the surface shall be a minimum.

$$S=\int\int\Big[(x_1-x)^2+(y_1-y)^2\Big]^n\,dxdy.$$

For minimum—

$$\frac{dS}{dx_1}=2n\int\int\Big[(x_1-x)^2+(y_1-y)^2\Big]^{n-1}(x_1-x)dxdy=0\ \ldots\ldots\ldots\ldots(1),$$

$$\frac{dS}{dy_1}=2n\int\int\Big[(x_1-x)^2+(y_1-y)^2\Big]^{n-1}(y_1-y)dxdy=0\ \ldots\ldots\ldots\ldots(2).$$

(1) and (2) may be satisfied in several ways.

(*A*). The curve may be such that the integration in question performed over the surface reduce to zero.

(*B*). $\Big[(x_1-x)^2+(y_1-y)^2\Big]^{n-1}dxdy=0$, or,

$$\int\int\Big[(x_1-x)^2+(y_1-y)^2\Big]^{n-1}dxdy=C.$$

$$(C).\ \begin{cases}(x_1-x)dxdy=0, \text{ or} \int\int(x_1-x)dxdy=C_1\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots(3),\\ (y_1-y)dxdy=0, \text{ and} \int\int(y_1-y)dxdy=C_2\ \ldots\ldots\ldots\ldots\ldots\ldots(4).\end{cases}$$

We shall only consider (*C*), as it is the only one which leads to the consideration of the center of gravity.

From (3) and (4), $x_1 = \dfrac{C_1 + \int\int x dx dy}{\int dx dy}$, and $y_1 = \dfrac{C_2 + \int\int x dx dy}{\int dx dy}$.

Therefore (x_1, y_1) is the center of gravity only when C_1 and C_2 are zero. For this condition to be fulfilled the first member of (3) and (4) must be evidently zero. From (1) and (2) we see that this will be true generally only when $n-1=0$, *i. e.*, $n=1$. Hence there can be no *general* proposition except for the sum of the squares of the distances.

II. $u = \int\int \left[(x_1-x)^2 + (y_1-y)^2\right]^n dx dy.$

$$\frac{du}{dx_1} = 2n\int\int \left[(x_1-x)^2 + (y_1-y)^2\right]^{n-1} (x_1-x) dx dy = 0,$$

$$\frac{du}{dy_1} = 2n\int\int \left[(x_1-x)^2 + (y_1-y)^2\right]^{n-1} (y_1-y) dx dy = 0.$$

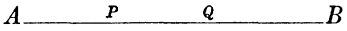
Whence $x_1 = \dfrac{\int\int \left[(x_1-x)^2 + (y_1-y)^2\right]^{n-1} x dx dy}{\int\int \left[(x_1-x)^2 + (y_1-y)^2\right]^{n-1} dx dy}$,

and $y_1 = \dfrac{\int\int \left[(x_1-x)^2 + (y_1-y)^2\right]^{n-1} y dx dy}{\int\int \left[(x_1-x)^2 + (y_1-y)^2\right]^{n-1} dx dy}$.

For (x_1, y_1) to be identical with center of gravity $n-1$ must be zero.

III. Prof. Zerr in his proof has $S = \int D dA$. He then writes $\dfrac{dS}{dx_1} = \dfrac{(x-x_1)dA}{D}$. He should have written $\dfrac{dS}{dx_1} = \int \dfrac{x-x_1}{D}. dA$; the integration was with respect to A and the differentiation with respect to x_1, and the two do not destroy each other.

IV. The proposition fails to hold for the simplest case imaginable, an indefinitely narrow rectangle, or straight line. Thus let AB be a straight line, P any point on that line. $AP=a$, $AB=l$, $PQ=x$. Then the sum of distances from



$P=S=\int_{-a}^{l-a}xdx=\frac{1}{2}[l^2-2al]$. $\frac{dS}{da}=-2l=0$ for minimum, *i. e.*, $l=0$ which is an absurdity. The sum of squares a minimum *will* hold in this case.

V. The same proof that Prof. Zerr gives will hold for *any* power of the distance, which proposition is highly improbable.

31. **Proposed by F. P. MATZ, D. Sc., Ph. D., Professor of Mathematics and Astronomy in Irving College, Mechanicsburg, Penn.**

In order that a vertical cylindric stalk may be severed by a blow of minimum force, the stalk must be struck at what inclination by a sharp wedge-shaped blade?

Solution by G. B. M. ZERR, A. M., Ph. D., Professor of Mathematics and Applied Science in Texarkana College, Texarkana, Ark.-Tex.

Let $f\phi(\theta)$=the force necessary to sever a unit of area, where θ is the inclination to the horizon. Let r=radius of stalk.

$\therefore$ The area of section made in cutting is $\pi r^2\sec\theta$, the area of an ellipse with semi-axes r and $r\sec\theta$. $\therefore \pi r^2 f\sec\theta\phi(\theta)$=a minimum. This can be made a minimum when $\phi(\theta)$ is known. If $f\phi(\theta)=a+b\cos^2\theta$, then $\theta=\sin^{-1}\sqrt{1=\frac{a}{b}}$.

32. **Proposed by S. H. WRIGHT, M. D., A. M., Ph. D., Penn Yan, New York.**

Intermittent reflections of flashes of light on a clear sky after dark, indicated a storm was progressing *below* the horizon. Refraction of 34′ on the horizon, brought the upper edge of the storm-cloud up to the horizon, and was just visible. How far off was the storm if the cloud was one mile above the earth?

I. Solution by the PROPOSER.

In the plane triangle ABC, let C be the center of the Earth, A the place of the observer, and B that of the cloud. Then AC=Earth's mean radius=3959 miles, $=b$, BC=3960 miles, $=a$, $AB=c$, the required distance. The angle BAC=the nadir distance of the cloud, being $90^\circ-34'=89^\circ26'=A$. Then

$\sin B=\frac{b\sin A}{a}$. $\therefore$ $B=88^\circ35'36''$, and $180^\circ-(A+B=1^\circ58'24''=C$, and

$$c=\frac{b\sin C}{\sin B}=\frac{a\sin C}{\sin A}=136.367 \text{ miles}.$$

II. Solution by G. B. M. ZERR, A. M., Ph. D., Professor of Mathematics and Applied Science in Texarkana College, Texarkana, Ark.-Tex.

Let A be the position of the observer, B the cloud, O the center of the earth, R=mean radius of the earth=3958 miles.

$$\therefore\ AC=2R\sin\tfrac{1}{2}AOC.\quad \angle ACB=\frac{\pi}{2}+\tfrac{1}{2}AOC,\quad \angle BAC=\tfrac{1}{2}AOC-34'.$$